



2019 Hong Kong Mathematics Kangaroo Contest

— Junior —

2019香港數學袋鼠競賽

【中學中年級】

Instruction 說明

1. DO NOT FLIP OPEN THIS FRONT COVER UNTIL YOUR PROCTOR TELLS YOU.
在未收到監考老師指示前，請不要翻開此封面。
2. This is a 30 question multiple choice test. For each question, only one answer choice is correct.
這是一套包括30道選擇題的測試，每道題目只有一個正確答案。
3. Each question is given a point value. You will receive full points for correct answer, and zero point for blank or incorrect answer. The full score of this test is 120 points.
每道題目都有給定的分值，答對得滿分，答錯或空白得0分。本次測試的滿分為120分。
4. Mark your answer to each problem on the answer form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be scored.
請將每道題目的答案用#2鉛筆標註在答題卡上。請注意檢查塗寫的黑色圓圈的準確性，用橡皮完全擦掉錯誤的答案和多餘的標記。只有恰當標註在答題卡上的答案才會被評分。
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
只能使用草稿紙、方格紙、尺、量角器和橡皮作為輔助工具。計算器是不允許使用的。測試中沒有任何問題必須需要使用計算器。
6. Figures are not necessarily drawn to scale.
圖形不一定按比例繪製。
7. Before beginning the test, make sure to record your name, school name and Competition ID on the answer form, especially to bubble in the 8-digit Competition ID completely!
在開始測試之前，請確保已將你的名字，校名和准考證號填寫在答題卡上，特別是8位准考證號的每位數字已經塗好相應的黑色圓圈。
8. You will have 75 minutes to complete the test once your proctor tells you to begin.
監考老師宣布開始後，你將有75分鐘的時間完成測試。

Part 1: 10 problems, 3 points each 第一部分：10道題目，每題3分
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1. $20 \times 19 + 20 + 19 =$

- (A) 389 (B) 399 (C) 409 (D) 419 (E) 429

2. A model train takes exactly 1 minute and 11 seconds for each round on a course. How long does it take for six rounds?

火車模型在一條軌道上轉一圈需要1分11秒。那麼轉六圈需要多少時間？

- (A) 6 minutes 56 seconds 6分56秒 (B) 7 minutes 6 seconds 7分6秒
 (C) 7 minutes 16 seconds 7分16秒 (D) 7 minutes 26 seconds 7分26秒
 (E) 7 minutes 36 seconds 7分36秒

3. A barber wants to write the word SHAVE on a board in such a way that a client looking into the mirror reads the word correctly. How should the barber write it on the board?

理髮師想要在板上寫一個單詞SHAVE，使得客人從鏡子裡看就能正確的讀出這個單詞。問理髮師應該如何寫？

- (A) **SHAVE** (B) **SHAVE** (C) **EVAHS**
 (D) **EVAH?** (E) **SHAVE**

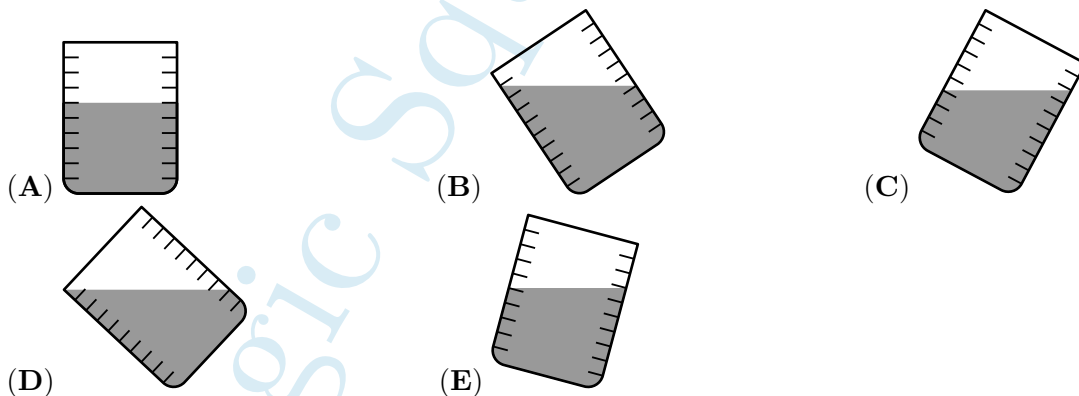
4. How many different sums of dots can you get by rolling three standard dice simultaneously?

同時拋擲三個標準的骰子所得到的點數之和有多少種不同的可能？

- (A) 14 (B) 15 (C) 16 (D) 17 (E) 18

5. Five identical glasses are filled with water. Four of them contain the same amount of water. Which one contains a different amount?

五個相同的玻璃杯裡都裝著水。其中四個杯子裡的水一樣多。問哪個杯子中的水與其他杯子裡的水不一樣？



6. A park has five gates. Monica wants to enter through one gate and to exit through a different one. In how many ways can she enter and exit the park?

一個公園有五個門。Monica想通過一個門進入並通過另一個門離開。問她有多少種方式進出公園？

- (A) 25 (B) 20 (C) 16 (D) 15 (E) 10

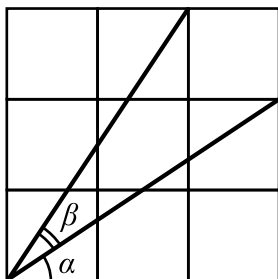
7. The weight of each of three kangaroos is a different whole number in kilograms. The total weight of them is 97 kilograms. How many kilograms can the lightest of them weigh at most?

三隻袋鼠的重量互不相同，都是整數千克。它們的總重量為97千克。問最輕的那隻袋鼠最多可能是多少千克？

- (A) 1 (B) 30 (C) 31 (D) 32 (E) 33

8. Which of the following statements is true for the marked angles in the given figure of nine identical squares?

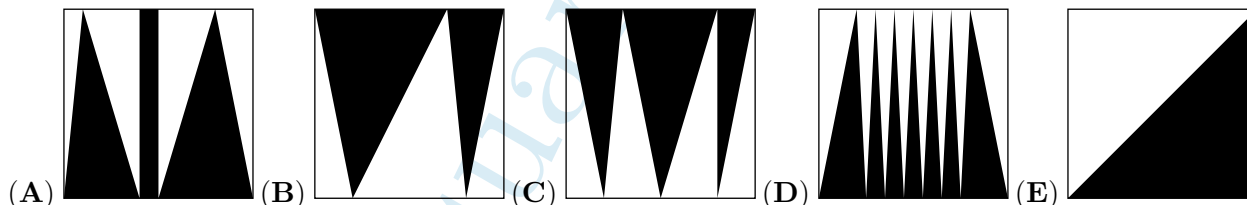
角度標記在給定的由九個正方形組成的圖形中，下列哪一項論斷是正確的？



- (A) $\alpha = \beta$ (B) $2\alpha + \beta = 90^\circ$ (C) $\alpha + \beta = 60^\circ$
 (D) $2\beta + \alpha = 90^\circ$ (E) $\alpha + \beta = 45^\circ$

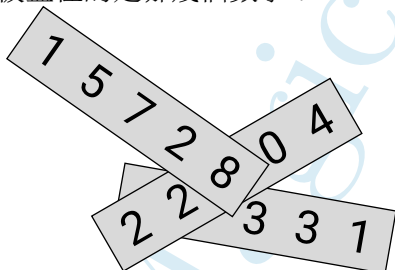
9. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

在每個單位正方形內部，都有一部分都塗成了黑色。哪個正方形中，黑色的總面積最大？



10. On each of three pieces of paper a five digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. Which are the covered digits?

如圖所示，在三張紙上分別寫有五位整數。有三個數字被覆蓋了。這三個數的總和是57263。問被蓋住的是哪幾個數字？



- (A) 0, 2, 2 (B) 1, 2, 9 (C) 2, 4, 9 (D) 2, 7, 8 (E) 5, 7, 8

Part 2: 10 problems, 4 points each | 第二部分：10道題目，每題4分

11. A square has vertices A, B, C, D labelled clockwise. An equilateral triangle is constructed with labels A, E, C labelled clockwise. What is the size of angle CBE in degrees?

一個正方形的頂點按順時針方向記為 A, B, C, D 。構造一個等邊三角形，使得沿順時針方向，它的三個頂點依次是 A, E, C 。問角 CBE 的大小是多少度？

- (A) 30 (B) 45 (C) 135 (D) 145 (E) 150

12. The numbers a, b, c, d are distinct positive integers chosen from 1 to 10. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?

數 a, b, c, d 是從 1 到 10 中選擇的不同正整數。 $\frac{a}{b} + \frac{c}{d}$ 可以取得的最小可能值是多少？


- (A) $\frac{3}{19}$ (B) $\frac{2}{10}$ (C) $\frac{14}{45}$ (D) $\frac{29}{90}$ (E) $\frac{25}{72}$


13. The flag of Kanguria is a rectangle with side lengths in the ratio 3 : 5. The flag is divided into four rectangles of equal area as shown. What is the ratio of the side lengths of the white rectangle?

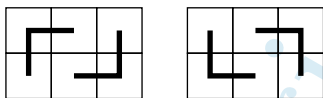
Kanguria 的旗幟是一個邊長比為 3 : 5 的矩形。如圖所示，這個旗幟被分成了四個面積相等的矩形。問白色矩形的邊長的比例是多少？



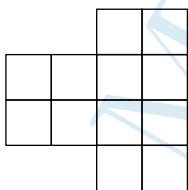
- (A) 1 : 3 (B) 1 : 4 (C) 2 : 7 (D) 3 : 10 (E) 4 : 15

14. A 3×2 rectangle can be exactly covered by two of the L-shape figures  in two different ways as shown below.

一個 3×2 的矩形可以有兩種不同的方式被兩個 L 形圖形  恰好完全的覆蓋。



In how many different ways can the figure below be covered by the L-shape figures?
問下圖有多少種不同的方式被 L 形圖形覆蓋？



- (A) 1 (B) 2 (C) 3 (D) 4 (E) 48

15. The triathlon consists of swimming, running, and biking. The biking is three-quarters of the total distance; the running is one-fifth; and the swimming is 2 kilometers. What is the total distance of this triathlon, in kilometers?

鐵人三項包括游泳，跑步和自行車。自行車的賽程是總賽程的四分之三；跑步的賽程是五分之一；游泳的賽程是2千米。問鐵人三項的總賽程是多少千米？

- (A) 10 (B) 20 (C) 38 (D) 40 (E) 60

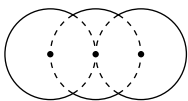
16. Some diluted juice is to be made out of concentrate and water in the ratio 1: 7 by volume. Juice concentrate is in a 1-litre flask, and the flask is half full. What fraction of this concentrate should be used to produce 2 litres of diluted juice?

稀釋果汁由濃縮果汁和水按1: 7的體積比製成。濃縮果汁在一個1升的容器中，容器是半滿的。應該使用這些濃縮果汁的幾分之幾來製作2升的稀釋果汁？

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{7}$ (D) $\frac{4}{7}$
(E) All of the concentrate. 全部濃縮果汁

17. The given shape is made of parts of three equal circles of radius R that have their centres on a straight line. The middle circle passes through the centres of the other two, as shown. What is the perimeter of the shape?

給定的形狀由三個相同的半徑為 R 的圓組成，三個圓的圓心都在一條直線上。如圖所示，中間的圓穿過另外兩個圓的圓心。問這個形狀的周長是多少？



- (A) $\frac{10\pi R}{3}$ (B) $\frac{5\pi R}{3}$ (C) $\frac{2\pi R\sqrt{3}}{3}$ (D) $2\pi R\sqrt{3}$ (E) $4\pi R$

18. The seven digits of the telephone number $\overline{aaabbbb}$ add up to the two digit number \overline{ab} . What is the sum $a + b$?

電話號碼 $\overline{aaabbbb}$ 中的七個數字相加的和是兩位數 \overline{ab} 。問 $a + b$ 是多少？

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

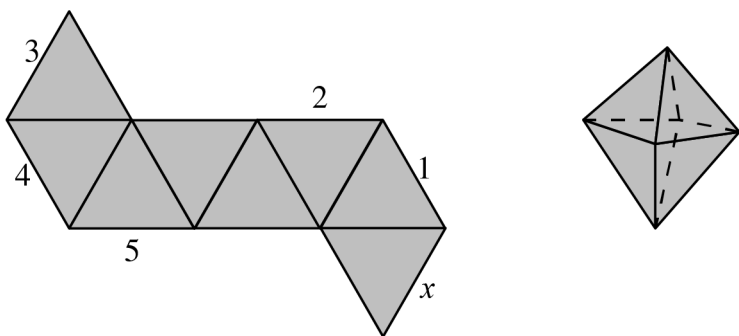
19. 60 apples and 60 pears are packed into boxes so that each box contains the same number of apples, and no two boxes contain the same number of pears. What is the largest possible number of boxes that can be packed in this way?

將60個蘋果和60個梨裝入盒子中，使得每個盒子中包含相同數量的蘋果，並且沒有兩個盒子中有相同數量的梨。問這樣最多能裝多少個盒子？

- (A) 20 (B) 15 (C) 12 (D) 10 (E) 6

20. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment marked with the x ?

下面是一個八面體的平面展開圖。當把它折成八面體時，用 x 標記的線段會和標記哪個數字的線段重疊在一起？

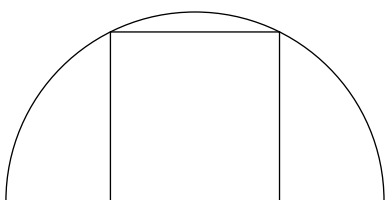


- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Part 3: 10 problems, 5 points each | 第三部分：10道題目，每題5分

21. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown. The radius of the circle is 1 cm. What is the area of the square, in cm^2 ?

如圖所示，正方形的兩個頂點位於半圓上，而另外兩個頂點位於半圓的直徑上。圓的半徑是1厘米。問正方形的面積是多少平方厘米？



- (A) $\frac{4}{5}$ (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{4}{3}$ (E) $\frac{2}{\sqrt{3}}$

22. Two dots are marked in a disc that is rotating around its centre. One of them is 3 cm further than the other to the centre of the disc and moves at a constant speed that is 2.5 times as fast as the other. What is the distance from the centre of the disc to this far point, in cm?

在一個圍繞其中心旋轉的圓盤上標記了兩個點。其中一個點距圓盤中心的距離比另一個點遠3厘米，並且這個點移動的恆定速度是另一個點的2.5倍。問較遠的點到圓盤中心的距離是多少厘米？

- (A) 10 (B) 9 (C) 8 (D) 6 (E) 5

23. The integers from 1 to 99 are written in ascending order without gaps. The sequence of digits is then divided into triplets of digits:

從1到99的整數按照升序沒有間隙的寫出。然後將這個數字序列分隔成三元數字組：

$$123456789101112 \dots 979899 \rightarrow (123)(456)(789)(101)(112) \dots (979)(899)$$

Which of the following is not one of the triplets?

以下哪一項不在這些三元數字組中？

- (A) (222) (B) (444) (C) (464) (D) (646) (E) (888)

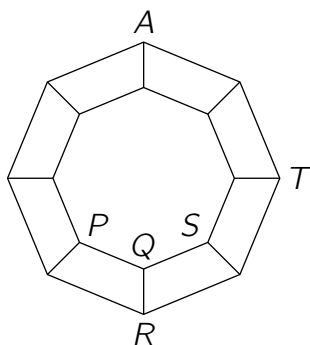
24. How many planes pass through exactly three vertices of a given cube?

有多少個平面通過給定立方體的恰好三個頂點？

- (A) 1 (B) 2 (C) 4 (D) 8 (E) 12

25. A graph consists of 16 vertices and some edges that connect them, as in the picture. An ant is now at the vertex labelled A . At each move, it can walk from one vertex to any neighbouring vertex crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 moves?

如圖所示，一個圖由16個頂點和一些連接頂點的邊組成。螞蟻現在位於標記為 A 的頂點處。在每次移動時，它可以從一個頂點爬到有邊相連的任何相鄰頂點。經過2019次移動，螞蟻有可能停留在圖中標記為 P, Q, R, S, T 的哪些頂點？



- (A) only P, R or S , not Q and T 只可能是 P, R 或 S ，但不可能是 Q 和 T
 (B) only P, R, S or T , not Q 只可能是 P, R, S 或 T ，但不可能是 Q
 (C) only Q 只可能是 Q
 (D) only T 只可能是 T
 (E) all of these are possible 這些頂點都可能

26. The positive integers a, b , and c each have three digits, and for each integer the first digit is the same as its last digit. Also $b = 2a + 1$ and $c = 2b + 1$. How many possibilities are there for the integer a ?

正整數 a, b 和 c 都是三位數，並且對於每個整數，其首位數字和末位數字相同。另外， $b = 2a + 1$ ，以及 $c = 2b + 1$ 。問整數 a 的取值有多少種可能？

- (A) 0 (B) 1 (C) 2 (D) 3
 (E) more than 3 多於3

27. On each vertex of a square, one positive integer is placed. For any two numbers joined by an edge of the square, one is a multiple of the other. However, for any two diagonally opposite numbers, neither is a multiple of the other. What is the smallest possible sum of the four numbers?

在正方形的每個頂點上，放置一個正整數。對於有正方形邊連接的任何兩個數，一個數是另一個數的倍數。但是對於相對的角上的兩個數，他們之間沒有整倍數關係。問這四個數之和的最小可能值是多少？

- (A) 12 (B) 24 (C) 30 (D) 35 (E) 60

28. $\{10, 20, 30, 40, 50, 60, 70, 80, 90\}$

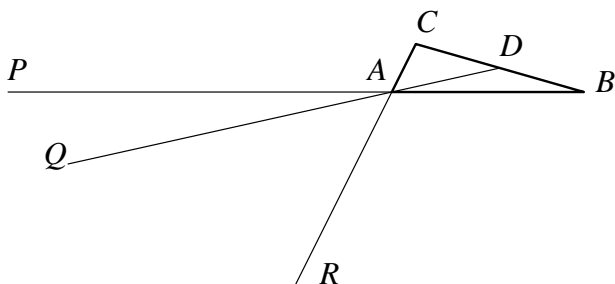
What is the least number of elements we have to delete from the above set so that the product of the elements remaining in the set is a perfect square?

我們必須從上面的集合中去掉最少幾個元素，使得剩下的所有元素的乘積是一個完全平方數？

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

29. Given triangle ABC of area S , let D be the midpoint of BC . Take points P, Q, R on lines AB, AD, AC , respectively, as shown in the picture, and such that $AP = 2 \cdot AB$, $AQ = 3 \cdot AD$ and $AR = 4 \cdot AC$.

給定面積為 S 的三角形 ABC ，設 D 為 BC 的中點。如果所示，分別在直線 AB, AD, AC 上選取點 P, Q, R ，滿足 $AP = 2 \cdot AB$ ， $AQ = 3 \cdot AD$ 和 $AR = 4 \cdot AC$ 。



What is the area of triangle PQR ?

問三角形 PQR 的面積是多少？

- (A) S (B) $2S$ (C) $3S$ (D) $\frac{1}{2}S$
 (E) 0 (i.e. P, Q, R are collinear 即 P, Q, R 共線).

30. If any digit of a given 4-digit number is deleted, the resulting 3-digit number is a divisor of the original number. How many 4-digit numbers have this property?

刪除給定四位數中的任何一個數字，得到的三位數都是原數的約數。問這樣的四位數有多少個？

- (A) 5 (B) 9 (C) 14 (D) 19 (E) 23